NAVAL POSTGRADUATE SCHOOL
Monterey, California

AD-A208 484

THESIS

SPIN STABILIZATION OF THE ORION SATELLITE
USING A THRUSTER
ATTITUDE CONTROL SYSTEM WITH OPTIMAL
CONTROL CONSIDERATIONS

by

Janet L. Cunningham

March 1989

Thesis Advisor   Harold A. Titus

89 6 05 117

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified		1b Restrictive Markings		
2a Security Classification Authority		3 Distribution Availability of Report		
2b Declassification Downgrading Schedule		Approved for public release; distribution is unlimited.		
4 Perf___ ng Organization Report Number(s)		5 Monitoring Organization Report Number(s)		
6a Nan. of Performing Organization Naval Postgraduate School	6b Office Symbol (if applicable) 39	7a Name of Monitoring Organization Naval Postgraduate School		
6c Address (city, state, and ZIP code) Monterey, CA 93943-5000		7b Address (city, state, and ZIP code) Monterey, CA 93943-5000		
8a Name of Funding Sponsoring Organization	8b Office Symbol (if applicable)	9 Procurement Instrument Identification Number		
8c Address (city, state, and ZIP code)		10 Source of Funding Numbers		
		Program Element No	Project No	Task No	Work Unit Accession No

11 Title (include security classification) SPIN STABILIZATION OF THE ORION SATELLITE USING A THRUSTER ATTITUDE CONTROL SYSTEM WITH OPTIMAL CONTROL CONSIDERATIONS

12 Personal Author(s) Janet L. Cunningham

13a Type of Report Master's Thesis	13b Time Covered From To	14 Date of Report (year, month, day) March 1989	15 Page Count 49

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes			18 Subject Terms (continue on reverse if necessary and identify by block number)
Field	Group	Subgroup	thesis, satellite, attitude control, spin stabilization.

19 Abstract (continue on reverse if necessary and identify by block number)

The controlled system is the ORION satellite spinning about its single axis of symmetry. Hydrazine thrusters are used as the control and are modeled by ideal, constant magnitude step functions.

The system is normalized and driven from non-zero initial angular velocities of the two axes other than the spin axis to the final condition of zero. The control profiles required to do this are determined based on a desired controller duty cycle. Adaptation of the duty cycle changes the ratio of the time the thrusters are on (fuel use) and total time to completion of the evolution.

A comparison between a single axis and a dual axis controller is presented. Simulation programs for the normalized system using a single axis controller, with a 100% duty cycle and a varying duty cycle, and a dual axis controller simulation program, with each controller having a duty cycle of no more than 50%, are developed.

The operation of the system is optimized using a system cost function. An equation relating the controller duty cycle of the dual system to the fuel/time trade-off parameter of the system cost function is required. A nonlinear feedback control algorithm (function of attitude angle rates) is developed from iterations of the simulation, and a priori knowledge of the form of the control from optimal control theory. This numerical solution will allow system designers to incorporate a closed form state feedback control for minimum fuel/time strategies using the ORION satellite's onboard software.

20 Distribution Availability of Abstract ☒ unclassified unlimited ☐ same as report ☐ DTIC users	21 Abstract Security Classification Unclassified	
22a Name of Responsible Individual Harold A. Titus	22b Telephone (include Area code) (408) 646-2560	22c Office Symbol 62Ts

DD FORM 1473,84 MAR   83 APR edition may be used until exhausted   security classification of this page
All other editions are obsolete

Spin Stabilization of the ORION Satellite Using a Thruster
Attitude Control System with Optimal Control Considerations

by

Janet L. Cunningham
Lieutenant Commander, United States Navy
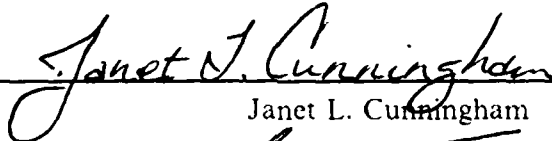B.S., Pennsylvania State University, 1978

Submitted in partial fulfillment of the
requirements for the degree of
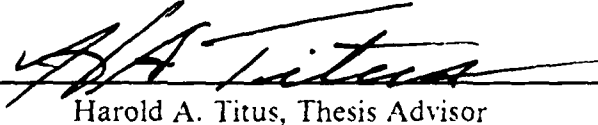
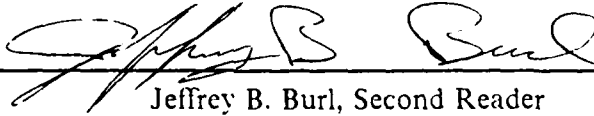MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
March 1989

Author: _____
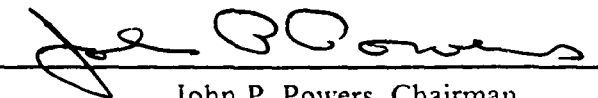Janet L. Cunningham

Approved by: _____
Harold A. Titus, Thesis Advisor

_____
Jeffrey B. Burl, Second Reader

_____
John P. Powers, Chairman,
Department of Electrical and Computer Engineering

_____
Gordon E. Schacher,
Dean of Science and Engineering

ABSTRACT

The controlled system is the ORION satellite spinning about its single axis of symmetry. Hydrazine thrusters are used as the control and are modeled by ideal, constant magnitude step functions.

The system is normalized and driven from non-zero initial angular velocities of the two axes other than the spin axis to the final condition of zero. The control profiles required to do this are determined based on a desired controller duty cycle. Adaptation of the duty cycle changes the ratio of the time the thrusters are on (fuel use) and total time to completion of the evolution.

A comparison between a single axis and a dual axis controller is presented. Simulation programs for the normalized system using a single axis controller, with a 100% duty cycle and a varying duty cycle, and a dual axis controller simulation program, with each controller having a duty cycle of no more than 50%, are developed.

The operation of the system is optimized using a system cost function. An equation relating the controller duty cycle of the dual system to the fuel/time trade-off parameter of the system cost function is required. A nonlinear feedback control algorithm (function of attitude angle rates) is developed from iterations of the simulation, and a priori knowledge of the form of the control from optimal control theory. This numerical solution will allow system designers to incorporate a closed form state feedback control for minimum fuel/time strategies using the ORION satellite's onboard software. *Keyw ds:* *pin stab... .., Satellite attitude co... ... systems. Turbo, ...) A—*

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

TABLE OF CONTENTS

vi

LIST OF TABLES

LIST OF FIGURES

I. INTRODUCTION

For the last three years a considerable amount of space systems research at the Naval Postgraduate School has been devoted to designing a small general purpose satellite (ORION).

The attitude control configuration chosen for the ORION prototype is spin stabilization. Other attitude control options, three-axis and gravity gradient stabilization, were considered for application on the ORION. The pointing accuracy for the gravity gradient stabilization is not highly refined. Although three-axis stabilization can achieve high pointing accuracy, it has problems with regard to thermal control, fuel consumption and requires a more complex sensor system. [1 : p. 466-479]

We apply the theory of fuel time optimized control to a spinning satellite, where a single axis of symmetry is assumed. The satellite spins about its axis of symmetry. Control, via hydrazine thrusters, maintains or drives the satellite to a zero spin (or angular velocity) about each of the two axes orthogonal to the axis of symmetry.

The purpose is to further explore an improved application of spin stabilization to the ORION satellite in an effort towards greater fuel efficiency and cost effectiveness.

II. SPIN STABILIZATION THEORY

A. TWO DIMENSIONAL RIGID BODY

A simple two dimensional rigid body model serves as the basis for developing the necessary equations of motion for the ORION satellite. Figure 1 portrays such a model in which the moment of inertia, I , is defined as:

$$I = ml^2 \tag{1}$$

where m is the particle mass and l is the moment arm length. The angular momentum, h, relative to the point O is expressed as:

$$h = I\omega = I\dot{\theta} \tag{2}$$

where ω is the angular velocity and is equal to $\dot{\theta}$, the rate of change of the angle θ. The moment of momentum, M, of the force, F, about point O is:

$$M = Fl \sin \theta \tag{3}$$
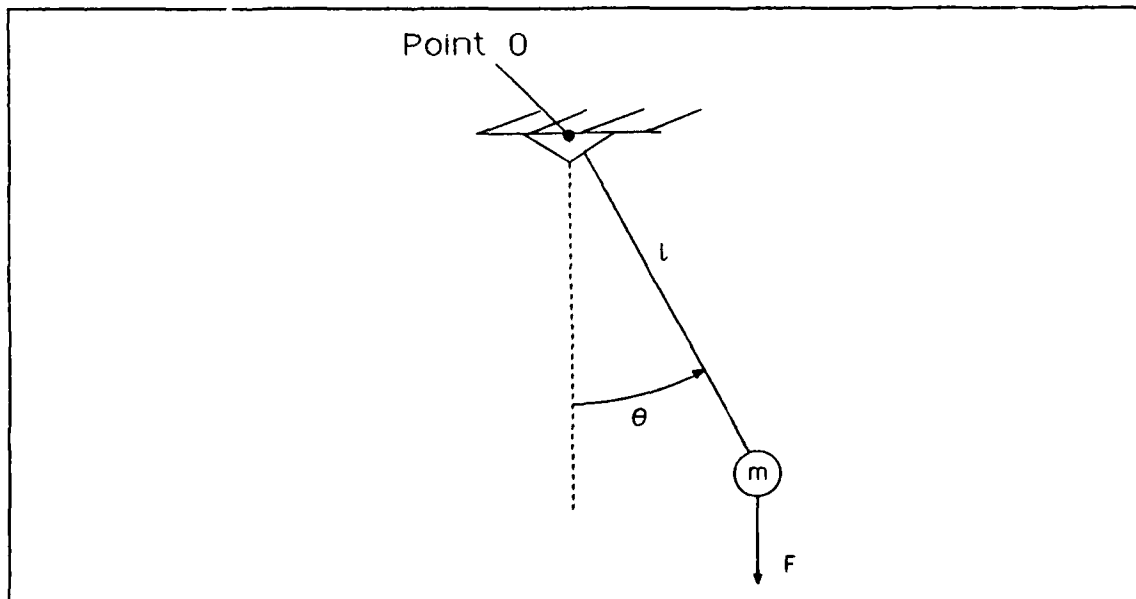
Figure 1.   Two Dimensional Rigid Body Diagram

2

This simple two dimensional model clearly states that momentum is a function of only three variables. However, the situation becomes more complicated when a three dimensional model is considered.

B. THREE DIMENSIONAL RIGID BODY

A three dimensional model is shown in Figure 2 with the three orthogonal components of angular momentum h_x, h_y and r. Rotation about any one of the three axes will produce a rotation through some angle. For example, rotation through an infinitesimal angle, $\Delta\phi$, about the the x-axis will result in two infinitesimal components of angular momentum: $h_y\Delta\phi\hat{k}$ and $h_z\Delta\phi(-\hat{j})$, where $\hat{j}$, $\hat{k}$ and $\hat{i}$ are unit vectors in the y, z and x axes, respectively. Likewise rotating the y-axis through an angle $\Delta\theta$ results in: $h_z\Delta\theta\hat{i}$ and $h_x\Delta\theta(-\hat{k})$. And finally, rotating the z-axis through an angle $\Delta\psi$ produces: $h_x\Delta\psi\hat{j}$ and $h_y\Delta\psi(-\hat{i})$. The infinitesimal changes in the original angular momentum components, $\Delta h_x\hat{i}$, $\Delta h_y\hat{j}$ and $\Delta h_z\hat{k}$ must also be considered.
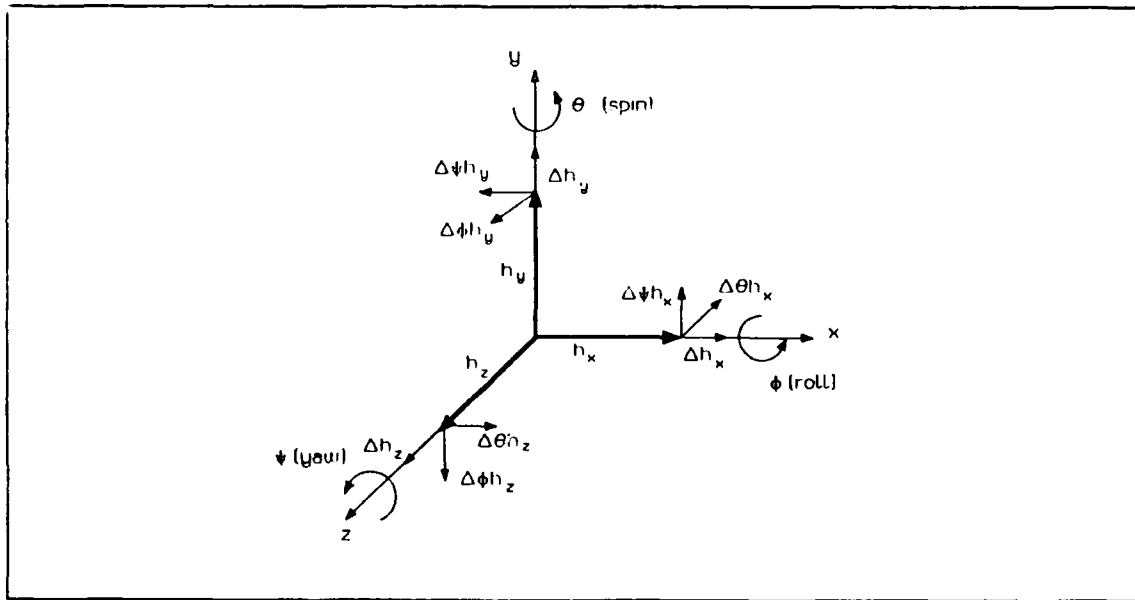
Figure 2.   Three Dimensional Rigid Body Diagram

Recalling that the moment of momentum, or torque, is equivalent to the rate of change of the angular momentum, then:

$$M = \lim_{\Delta t \to 0} \frac{\Delta h}{\Delta t} \tag{4}$$

3

Adding the components described in the preceding paragraph and taking the limit as Δt goes to zero, results in the following momentum equations:

$$M_x = \dot{h}_x - h_y \dot{\psi} + h_z \dot{\theta}$$
$$M_y = \dot{h}_y - h_z \dot{\phi} + h_x \dot{\psi} \qquad (5)$$
$$M_z = \dot{h}_z - h_x \dot{\theta} + h_y \dot{\phi}$$

Applying Equation 2 to a three dimensional body gives [2: p. 109]:

$$\begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad (6)$$

If the products of inertia are ignored [3: p. 51], such that $I_{ij} = 0$ for $i \neq j$ for i and $j \in \{x, y, z\}$ then Equation 6 reduces to:

$$h_x = I_{xx} \dot{\phi}$$
$$h_y = I_{yy} \dot{\theta} \qquad (7)$$
$$h_z = I_{zz} \dot{\psi}$$

The momentum equations can now be written as:

$$M_x = I_{xx} \ddot{\phi} - I_{yy} \dot{\theta} \dot{\psi} + I_{zz} \dot{\psi} \dot{\theta}$$
$$M_y = I_{yy} \ddot{\theta} - I_{zz} \dot{\psi} \dot{\phi} + I_{xx} \dot{\phi} \dot{\phi} \qquad (8)$$
$$M_z = I_{zz} \ddot{\psi} - I_{xx} \dot{\phi} \dot{\theta} + I_{yy} \dot{\theta} \dot{\phi}$$

In the case of a rigid body satellite, where external moments are absent [4: p. 524], Equation 8 can be simplified by rearranging to become:

$$I_{xx} \ddot{\phi} = (I_{yy} - I_{zz}) \dot{\theta} \dot{\psi}$$
$$I_{yy} \ddot{\theta} = (I_{zz} - I_{xx}) \dot{\phi} \dot{\psi} \qquad (9)$$
$$I_{zz} \ddot{\psi} = (I_{xx} - I_{yy}) \dot{\theta} \dot{\phi}$$

If the satellite spins uniformly around the pitch, θ, axis (also called the spin and/or y axis) with an angular velocity of w_s and assuming the satellite has a single axis of symmetry, we can conclude that $I_{xx} = I_{zz}$. We will also assume that $\dot{\theta} = w_s + \varepsilon$, where ε, the spin error, is a small angular velocity error due to perturbation and that $\varepsilon \neq 0$, $\dot{\phi} \neq 0$, and $\dot{\psi} \neq 0$, with $\dot{\phi}$ and $\dot{\psi}$ both small. [2: p. 114]

4

Incorporating all of these assumptions into the euler equations, they now become:

$$\ddot{\phi} = \frac{I_{yy} - I_{xx}}{I_{xx}} w_s \dot{\psi}$$

$$I_{yy}\dot{\varepsilon} = 0 \tag{10}$$

$$\ddot{\psi} = \frac{I_{xx} - I_{yy}}{I_{xx}} w_s \dot{\phi}$$

$I_{yy}\dot{\varepsilon} = 0$ reveals that the rate of change of the spin error is zero and therefore the spin error is a constant! Normalize the system by letting:

$$t = (\frac{I_{xx}}{I_{yy} - I_{xx}}) \frac{\tau}{w_s} \tag{11}$$

The state variables are expressed in terms of the system parameters as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ \dot{\psi} \end{bmatrix} \tag{12}$$

Substituting Equation 11 into the euler equations and converting to state variable annotation results in:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{13}$$

Applying control inputs to the system, the following system equations are obtained:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{14}$$

where u_1 and u_2 are the normalized thruster control torques for the x_1 (roll) and x_2 (yaw) axes, respectively.

Initially the system will be investigated with u_2 set equal to zero and not used. Then both u_1 and u_2 will be used at the same time. The following chapter on optimal control theory will explore how the values for these two controllers are assigned.

III. OPTIMAL CONTROL

Ideal control is a theoretical concept that may not be achievable, since it does not account for the physical constraints of the system it is designed for. Conversely, optimal control takes the physical constraints of the system into consideration. When a control system is chosen based on a given satellite design and a set performance index, optimization is the result.

A. PERFORMANCE INDICES

In general, a performance index is a function of system parameters and to a large degree defines the character of the optimal control. This, in turn, determines the configuration of the control system.

A performance index is normally chosen based upon the system's requirements, which may often be in conflict with each other. For example, one requirement for a satellite system may be to maintain a specified orientation (within certain given limits of accuracy), while at the same time being required to maximize satellite lifetime. Since lifetime is an inverse function of fuel usage, maximizing lifetime means minimizing fuel usage. This is in direct conflict with the requirement for maintaining satellite orientation, which constantly requires fuel for adjustments. The greater the requirement for accuracy of satellite orientation, the greater the rate of fuel usage. Consequently, a performance index often represents a compromise of system requirements.

Changing performance indices further complicates matters, since change in performance indices results in a different optimal control. Practicality also enters the picture, since a desired optimal control may be beyond the capabilities of readily available hardware, making the desired optimal control system impractical.

The difficulty of choosing an appropriate performance index for a complicated system is further compounded by the fact that practicality requires the components of a performance index be easily measured or computed through sensors or rate gyros. In choosing a performance index, experience has shown that preference should be given to the index that is developed from an application rather than one developed from a pure mathematical point of view.

An appropriate control must be chosen which will minimize the performance index, J, which is a cost function that is defined as:

6

$$J = f\{\mathbf{x}(t), t_o, t_f, \mathbf{u}(t)\} \tag{15}$$

where $\mathbf{x}(t)$ is the state vector (Equation 12), t_o and t_f are the initial and final times of the system's operation, and $\mathbf{u}(t)$ refers to the control vector (Equation 14).

It is important to understand that, in the optimal control application given in this paper, J is a function of the control input, $\mathbf{u}(t)$, and the control input is itself a function of $\mathbf{x}$. That is, the state vector, $\mathbf{x}(t)$, is included as a parameter for J since $\mathbf{x}(t)$ has such a great influence upon the control input parameter, $\mathbf{u}(t)$.

The time optimal control system is one of two performance indices to be considered, and is defined as:

$$J = \int_{t_o}^{t_f} dt \tag{16}$$

where the time interval $t_o \le t \le t_f$ is finite. The second performance index to be considered is the fuel optimal control system:

$$J = \int_{t_o}^{t_f} \sum_{i=1}^{2} |u_i(t)| \, dt \tag{17}$$

For the purpose of this discussion, $\mathbf{u}(t)$ is defined as the thruster's state at a specified time t. Three states will be considered: "off", "on" (positive), and "on" (negative). The positive or negative condition for "on" state is an important consideration, as will be seen.

Combining Equations 16 and 17 results in the performance index to be implemented in this study. This index is defined as:

$$J = \int_{t_o}^{t_f} 1 + \lambda \sum_{i=1}^{2} |u_i(t)| \, dt \tag{18}$$

where λ is a weighting factor that influences the compromise between control response and fuel use.

The weighting factor, λ, is a critical parameter that is constant and positive in value. By setting λ equal to zero, Equation 18 becomes a simple minimum time problem. If λ

7

is increased to a value approaching infinity, then we have a minimum fuel problem. Determining a performance measure for our system reduces to that of determining one parameter: λ. The following sections which simulate the normalized system will explore this in more detail.

B. SINGLE VS DUAL CONTROL

1. One Controller

a. Minimum Time

The minimum time switching curve for a spinning satellite is the next topic to be considered. When dealing with a control system where only one controller is allowed to operate (i.e., u_2 is set equal to zero), u_1 is a function of both states x_1 and x_2. When x_1 versus x_2 is plotted, regions for the values of u_1 can be described. This plot is called the state space or phase plane. In our minimum time problem, where λ is zero, u_1 is always turned on. This is the case for an ideal thruster with a non-varying thrust magnitude, $|u_1|$ = constant. The thruster is always turned on. It can cause the satellite body to rotate in a positive or negative direction since the sign of u_1 can be positive or negative. For simplicity, a magnitude of unity for this constant normalized torque will be used. The true switching curve for these conditions is given in Figure 3 [5: p. 29.]. Since this is difficult to model, the switching curve described in Figure 4 is often used. Consequently, the curve in Figure 4 will be used in this study when simulating the system model when there is only one "on" controller.

If the minimum time system is started at the initial condition: $x_1 = 6.844$ and $x_2 = -6.844$, then the plot given in Figure 5 shows the system's response. All of the half circles above the x-axis have origins at the point $x_1 = -1$, $x_2 = 0$. Likewise all of the half circles below the x-axis have origins at the point $x_1 = +1, x_2 = 0$. This is not a coincidence.

In Figure 4, regressing in time from the origin allows the system to follow either of the cusps. The right cusp is chosen ($u_1 = +1$), and followed from the origin until the x-axis is approached at the point $x_1 = +2$ and $x_2 = 0$. In other words, a half circle is drawn counterclockwise (reverse time) around the point $x_1 = +1, x_2 = 0$ starting from the origin to the point $x_1 = +2$ and $x_2 = 0$. When this point is crossed, the switching curve boundary is also crossed and the controller changes from $u_1 = +1$ to $u_1 = -1$, and a new arc must be drawn (counterclockwise, as before) starting at the point $x_1 = +2$ and $x_2 = 0$ with a new origin at the point $x_1 = -1$ and $x_2 = 0$. This arc continues until it becomes a half or semicircle at the point $x_1 = -4$ and $x_2 = 0$. Continuing in the same

8
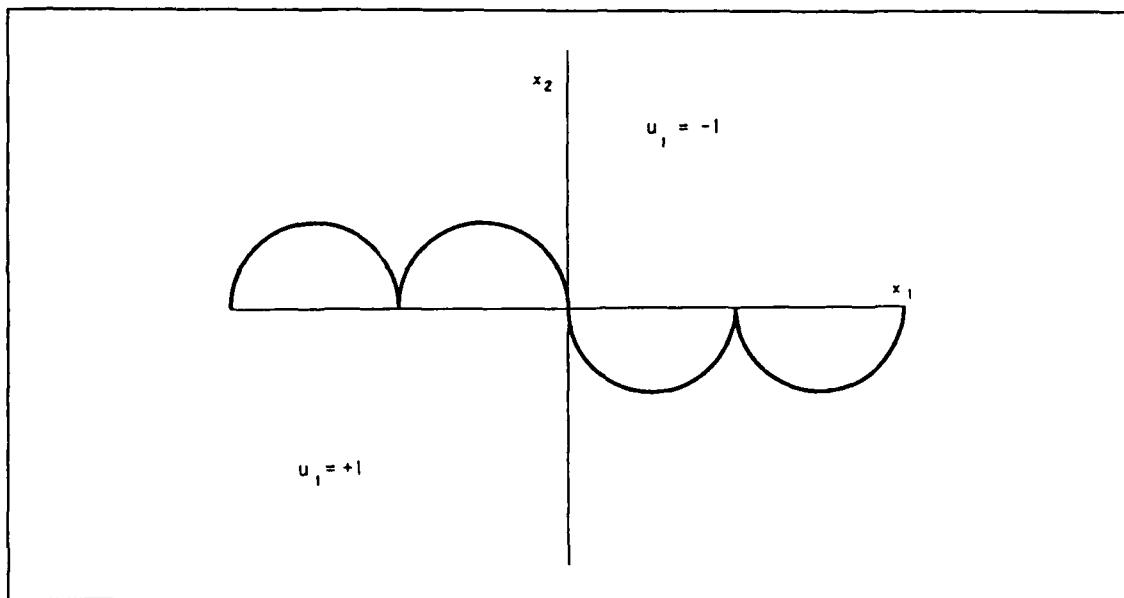
Figure 3.   True Minimum Time Switching Curve for One Controller
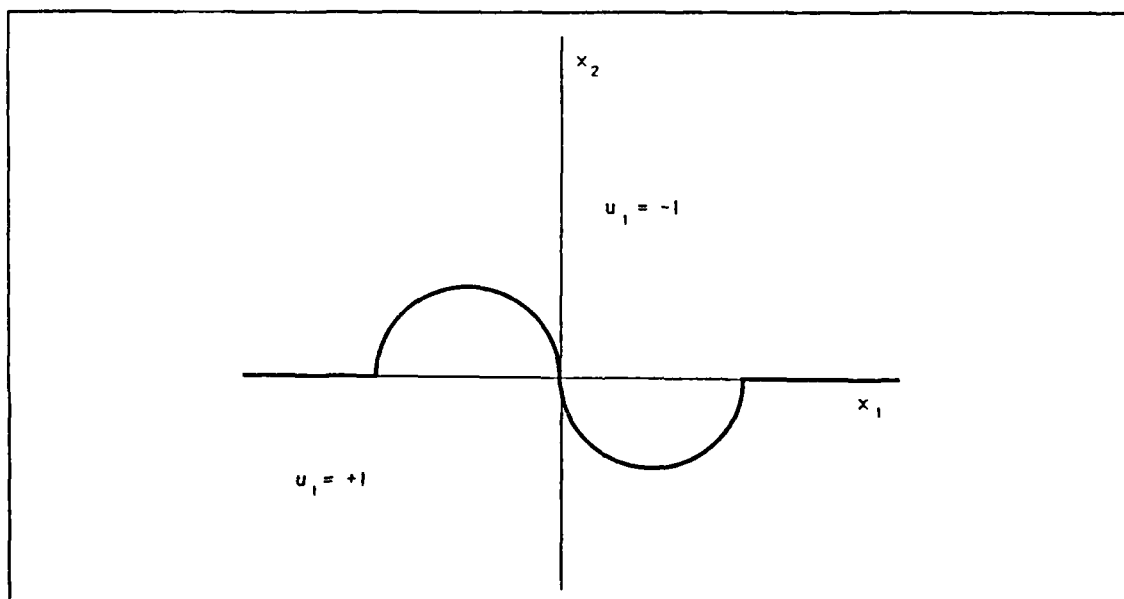
Figure 4.   Approximated Minimum Time Switching Curve for One Controller

manner such that two semicircles are drawn above the x-axis and two plus $\frac{180 - \alpha}{180}$, where $\alpha = \tan^{-1}\frac{6.844}{5.844} = 49.506$ degrees, semicircles are drawn below the x-axis. By simple geometry the final point for this reverse time exercise is the same point as the

9
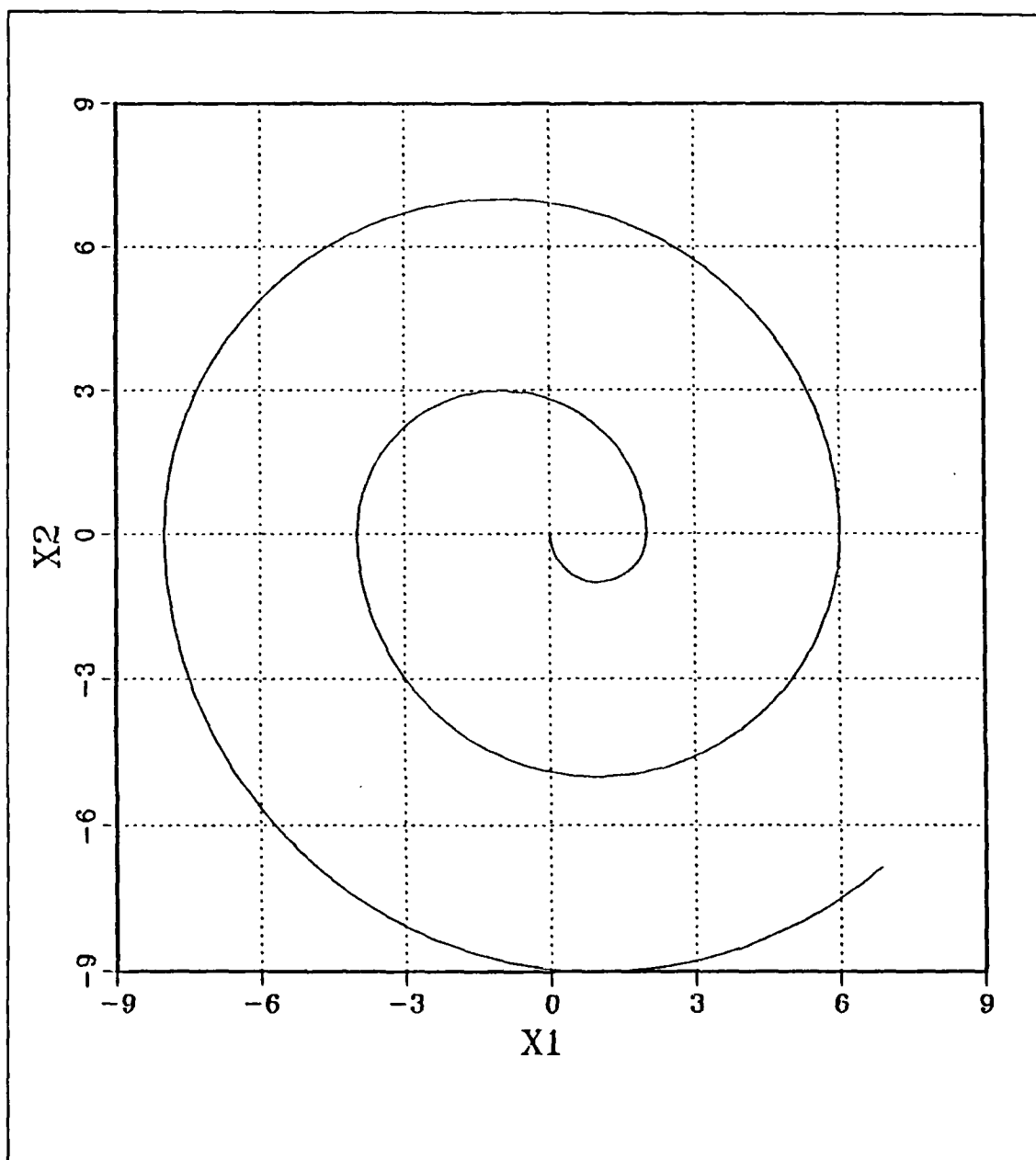
Figure 5. System State Space with One Minimum Time Controller: Initial Condition (6.844,-6.844)

initial condition for the minimum time (forward time, clockwise) system simulation. The conclusion that can be made from this is that the system is modeled correctly.

Another check can be employed to ensure this even further. Since the system is normalized, the time it takes to travel each half circle corresponds to π seconds. Therefore in this system model simulation, it is expected that it would take 4.7249 times π seconds (14.84 seconds) to go from the initial condition to the zero condition. The computer simulation takes 14.82 seconds to get to the point $x_1 = -.000000618$, $x_2 = -.0005524$. Due to the configuration of the computer program, driving the system to zero results in iterative reducing of the integration step size of the simulation so that much computer time is wasted. This "close enough" final condition strongly implies that the system model is running as expected.

b. *Minimum Fuel/Time*

The discussion above dealt with the minimum time problem, where λ is set equal to zero. Now it must be determined how to model the system when the controller can be "off", i.e., $u = 0$ and therefore $\lambda \neq 0$. The following paragraphs develop insight in the minimum fuel time control problem using Pontryagin's Minimum Principle [6] for second order linear systems with bounded control inputs.

Recalling that the system in state variable annotation can be written in the form of first-order linear differential equations:

$$x_i = f_i\{\mathbf{x}, \mathbf{u}\} \tag{19}$$

where $\mathbf{x}$ is the vector of the state variables (x_1, x_2), and $\mathbf{u}$ is the vector of control variables (u_2, u_1).

The Hamiltonian defined as:

$$H = J_{\text{arg}} + \sum_{i=1}^{2} p_i f_i\{\mathbf{x}, \mathbf{u}\} \tag{20}$$

must be minimized. Here J_{arg} is the argument $(1 + \lambda(|u_1| + |u_2|))$ of the cost function, J. Also p_1 and p_2 are auxilliary variables given by:

$$\frac{dp_i}{dt} = \frac{-\delta H}{\delta x_i} \tag{21}$$

and,

$$\frac{dx_i}{dt} = \frac{\delta H}{\delta p_i} \tag{22}$$

11

Substituting Equation 14, Equation 20 becomes:

$$H = p_1(x_2 + u_2) + p_2(-x_1 + u_1) + 1 + \lambda(|u_1| + |u_2|) \tag{23}$$

Substituting Equation 23 into Equation 21 and solving gives:

$$\dot{p}_1 = +p_2$$
$$\dot{p}_2 = -p_1 \tag{24}$$

The solution to Equation 24 is of the form:

$$p_1 = A \sin(t - \phi)$$
$$p_2 = A \cos(t - \phi) \tag{25}$$

where A and ϕ are integration constants.

To minimize the Hamiltonian, the form of the optimal control is:

$$
\begin{aligned}
u_1 &= -\text{sign}(p_2) \quad && \text{if } |p_2| > \lambda \\
u_1 &= 0 \quad && \text{if } |p_2| \leq \lambda \\
u_2 &= -\text{sign}(p_1) \quad && \text{if } |p_1| > \lambda \\
u_2 &= 0 \quad && \text{if } |p_1| \leq \lambda
\end{aligned}
\tag{26}
$$

The system with no control applied will describe a circle in its phase plane. In other words, if the system starts at some initial condition other than zero, the system state values will revolve clockwise (forward time) around the origin of the x_1, x_2 state space plot. Since no analytical solution was obtained for the minimum fuel time problem, an optimized system model is deduced. Based upon Equation 26 and the sinusoidal requirement described in Equation 25, both u_1 and u_2 must cycle from plus to minus every 180 degrees and be 90 degrees out of phase with each other. Two lines with slopes that have the same magnitude but are of opposite sign will achieve the proper periodicity requirement, since the regions where the control has a value of zero will always take the same period of time to travel through. Figure 6 shows lines with slopes of -1 and $+1$ ($\theta = 90$ degrees) and the minimum time switching curve. Since the value for u_1 above the switching curve is -1, and below is $+1$, there is a conflict in values for the controller in

the shaded regions A and B. In Table 1, the experimental matrix is shown where three simulations will be run with varying values for the single controller, u_1.

Table 1.   CONTROLLER ASSIGNMENT FOR THE SINGLE CONTROLLER

	Region A	Region B
Method 1	$u_1 = -1$	$u_1 = +1$
Method 2	$u_1 = +1$	$u_1 = -1$
Method 3	$u_1 = 0$	$u_1 = 0$
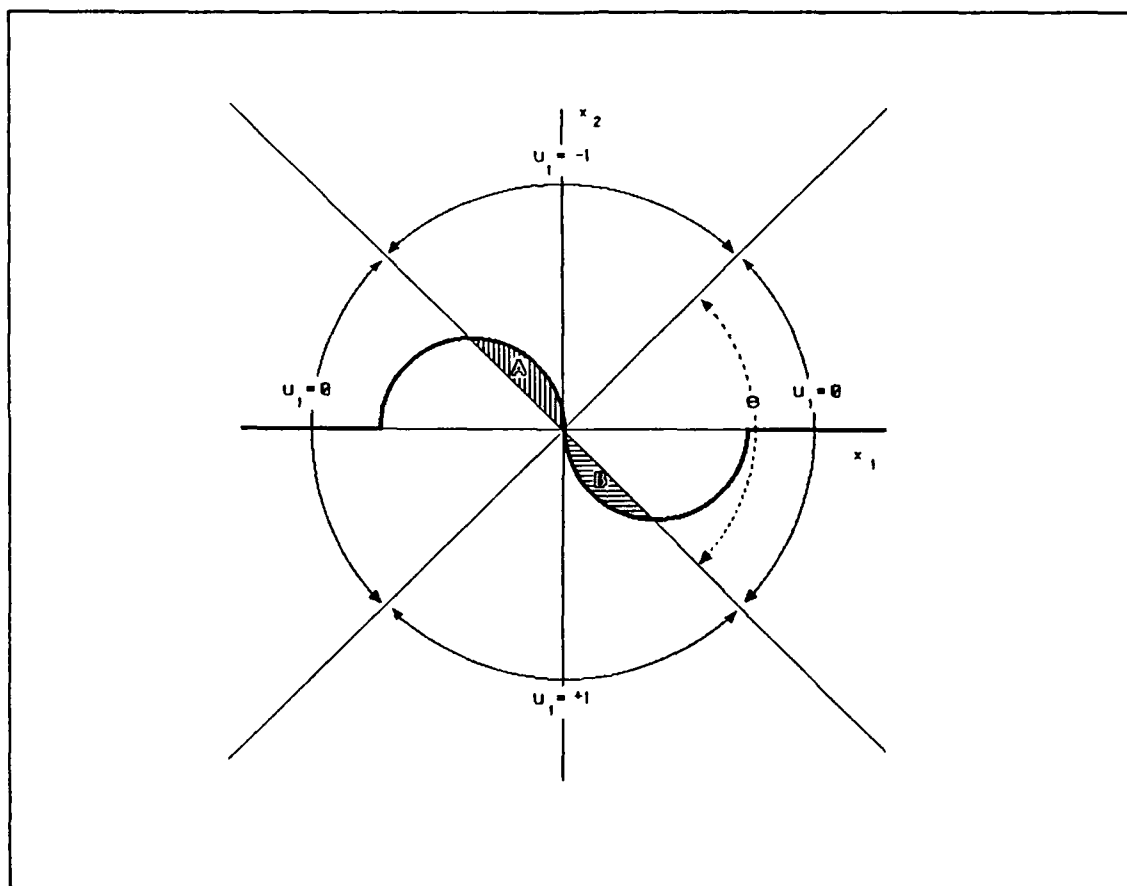
Figure 6.   Controller Assignment for One Controller

Figure 7 shows the curve that is common to all three methods. The final point (the one closest to the axes' origin) for the three simulations is just outside region B, as depicted in Figure 6. The separate plots of the three methods from their last common point to

13

an end condition described by a circle about the axes' origin with a radius of .01 are shown in Figure 8.

The results of the experimental matrix shown in Table 1 are presented in Table 2.

Table 2. SINGLE CONTROLLER RESULTS

	Time to End Condition	Fuel Consumed
Method 1	20.85	10.80
Method 2	20.17	10.75
Method 3	20.18	10.61

Figure 7. One Controller - Minimum Fuel/Time for the Far Field: Initial Condition (6.844,-6.844); $\theta = 90$ degrees
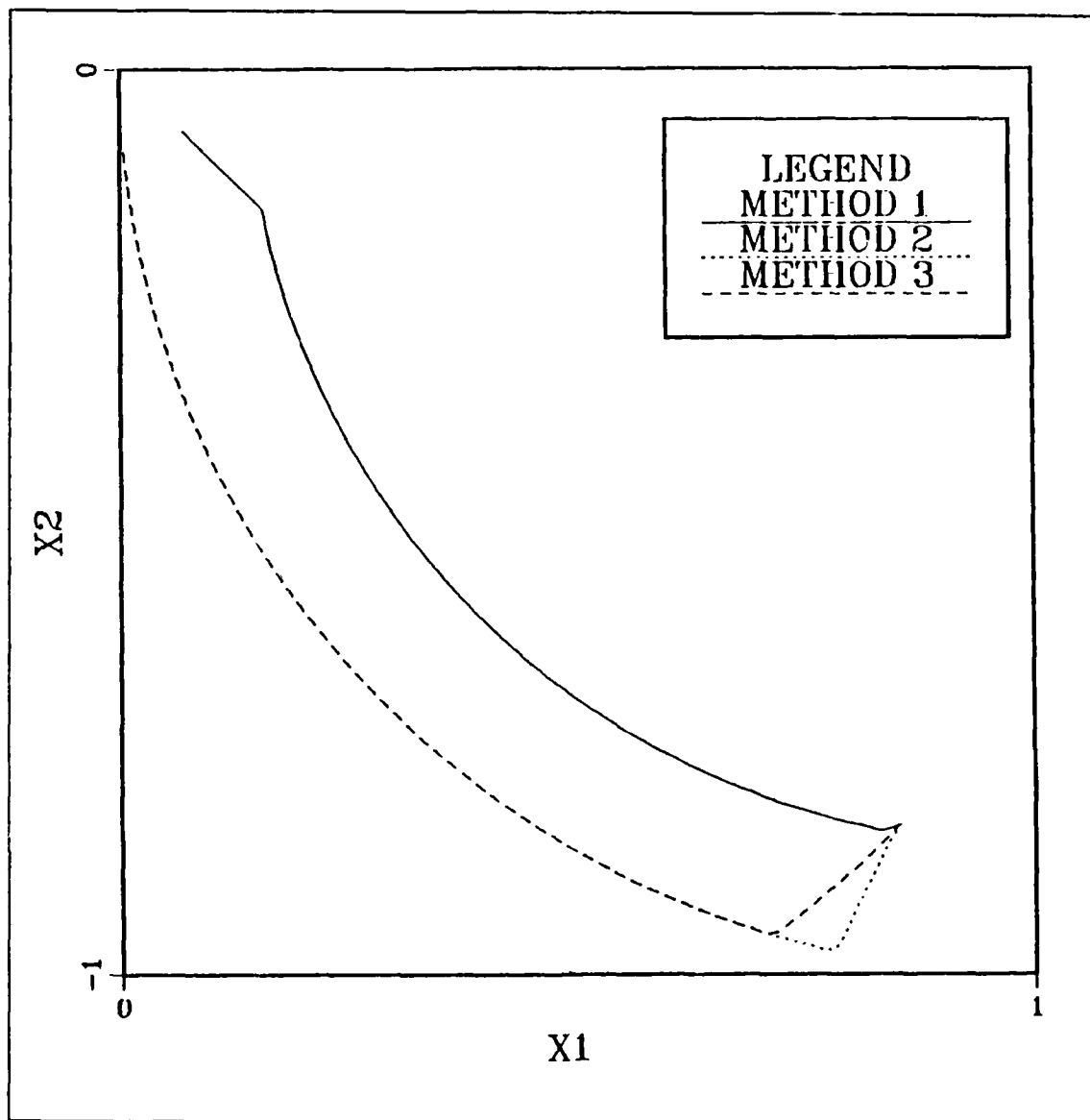
Figure 8. One Controller - Minimum Fuel/Time for the Near Field: $\theta = 90$ degrees

2. Two Controllers

a. *Minimum Time*

The switching curves for the minimum time dual control system are shown in Figure 9. The analytical derivation for these curves can be found in [7]. Again, as with the single minimum-time controller, these curves are difficult to model, so they are

linearized in the region outside a circle of radius two about the axes' origin. This curve is presented in Figure 10.
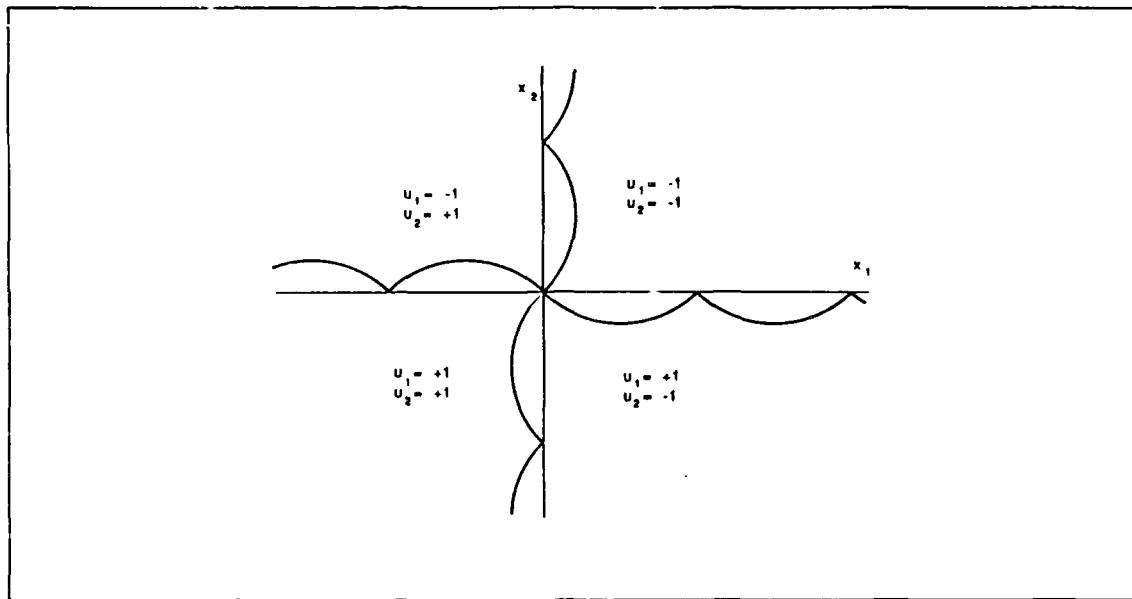
Figure 9.   True Minimum Time Switching Curve for Two Controllers
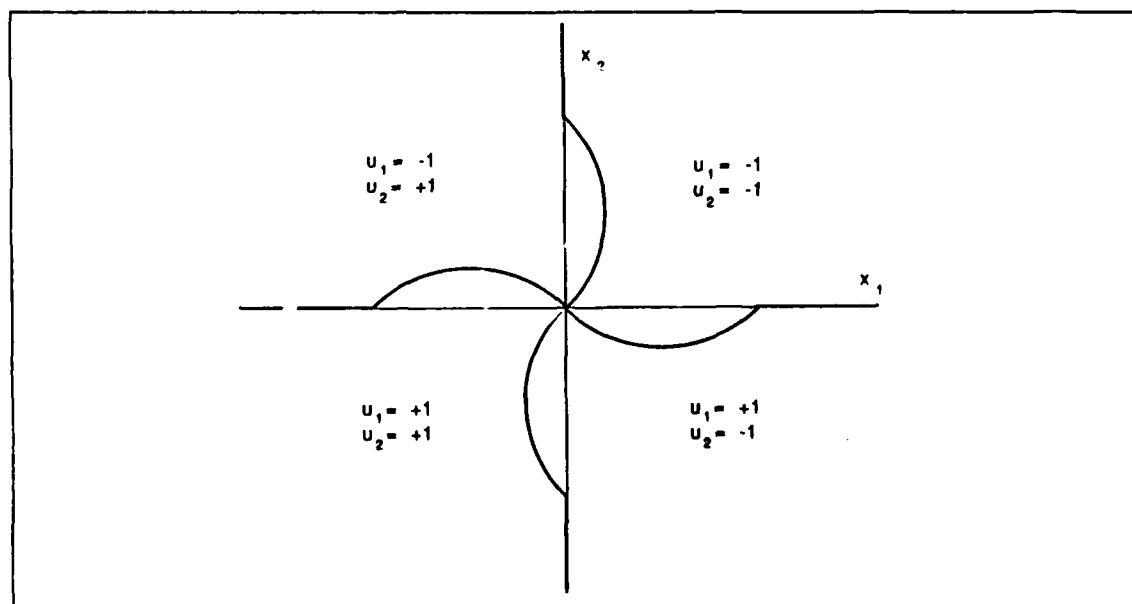
Figure 10.   Approximated Minimum Time Switching Curve for Two Controllers

b. *Minimum Fuel/Time*

The major difference between the switching curves for the dual control system and the one controller switching curve given in Figure 6 is that there is now a controller on the y-axis.

The control regions for the case where each of the two controllers has less than a 50% duty cycle (Case 1) is depicted in Figure 11. Figure 12 shows the switching curves and control areas when the angle θ, the deadzone angle, equals 90 degrees. From the state equations it can be seen that if $x_1 = 0$ then $0 = x_2 + u_2$, or $x_2 = -u_2$. In other words, when $x_2 = +1$ then $u_2 = -1$. Thus the point $(0, +1)$ in the phase plane plot equates to the controller value: $u_2 = -1$. Likewise, the point $(0, -1)$ corresponds to: $u_2 = +1$. This is a subtle but important point. Be aware that the switching curve origins for the controllers will be located on the x and y axes ONLY when $\theta \geq 90$ degrees. (This will not be the case when $\theta < 90$ degrees. Discussion of this alternative will appear later.)

Region 1 in Figure 12 has the following controller assignment $u_1 = -1$ and $u_2 = 0$; region 2 has $u_1 = +1$ and $u_2 = 0$; region 3 has $u_1 = 0$ and $u_2 = +1$; and region 4 has $u_1 = 0$ and $u_2 = -1$. The minimum time switching curve for the x-axis remains as it was for the one controller case; u_1 has the value of -1 above the curve and $+1$ below. The minimum time switching curve for the y-axis is the new addition. The value for u_2 is -1 to the right of this switching curve, and $+1$ to the left.

The four shaded areas in Figure 12 imply a controversial assignment of the controller values. Three simulations with varying values for the shaded regions a, b, c, and d will be run as described in the experimental matrix shown in Table 3.

Table 3. CONTROLLER ASSIGNMENT FOR THE DUAL CONTROLLER
($\theta = 90$ degrees)

	Region a $u_2 = 0$	Region b $u_2 = 0$	Region c $u_1 = 0$	Region d $u_1 = 0$
Method 1	$u_1 = -1$	$u_1 = +1$	$u_2 = +1$	$u_2 = -1$
Method 2	$u_1 = +1$	$u_1 = -1$	$u_2 = -1$	$u_2 = +1$
Method 3	$u_1 = 0$	$u_1 = 0$	$u_2 = 0$	$u_2 = 0$

Results from the series of simulations are shown in Figure 13 and Figure 14. In Figure 13 the curve that is common to all three methods is shown. The final point (the one closest to the axes' origin) is just outside the region depicted in

18

Figure 12 as region a. The plot for the three methods starting from their last common point to an end condition described by a circle of radius .01 around the axes's origin is given in Figure 14. Table 4 compares the results of these three methods.

Table 4. DUAL CONTROLLER RESULTS ($\theta = 90$ degrees)

	Time to End Condition	Fuel Consumed
Method 1	10.81	10.81
Method 2	10.75	10.75
Method 3	10.75	10.60
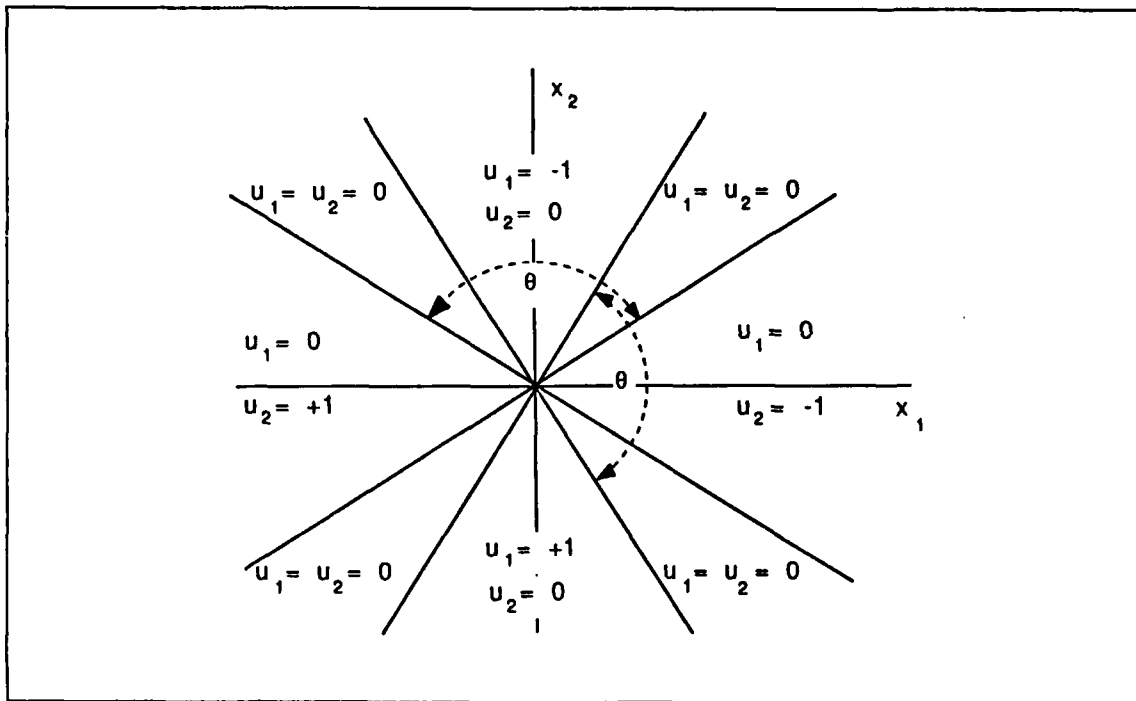
Figure 11.   Controller Assignment for Two Controllers - Case 1: $\theta \geq 90$ degrees

19
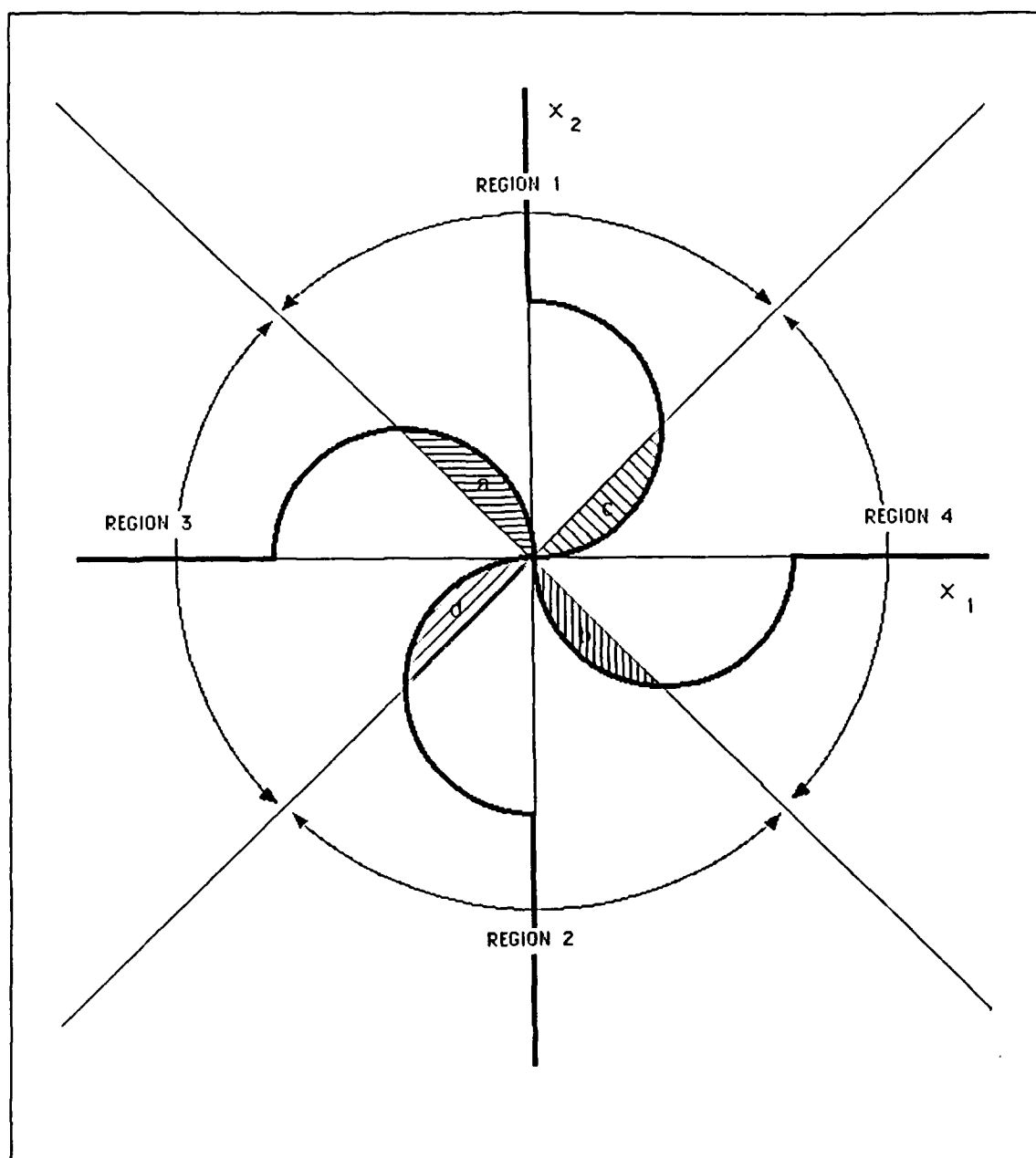
Figure 12.   System State Space with Two Minimum Fuel/Time Controllers:
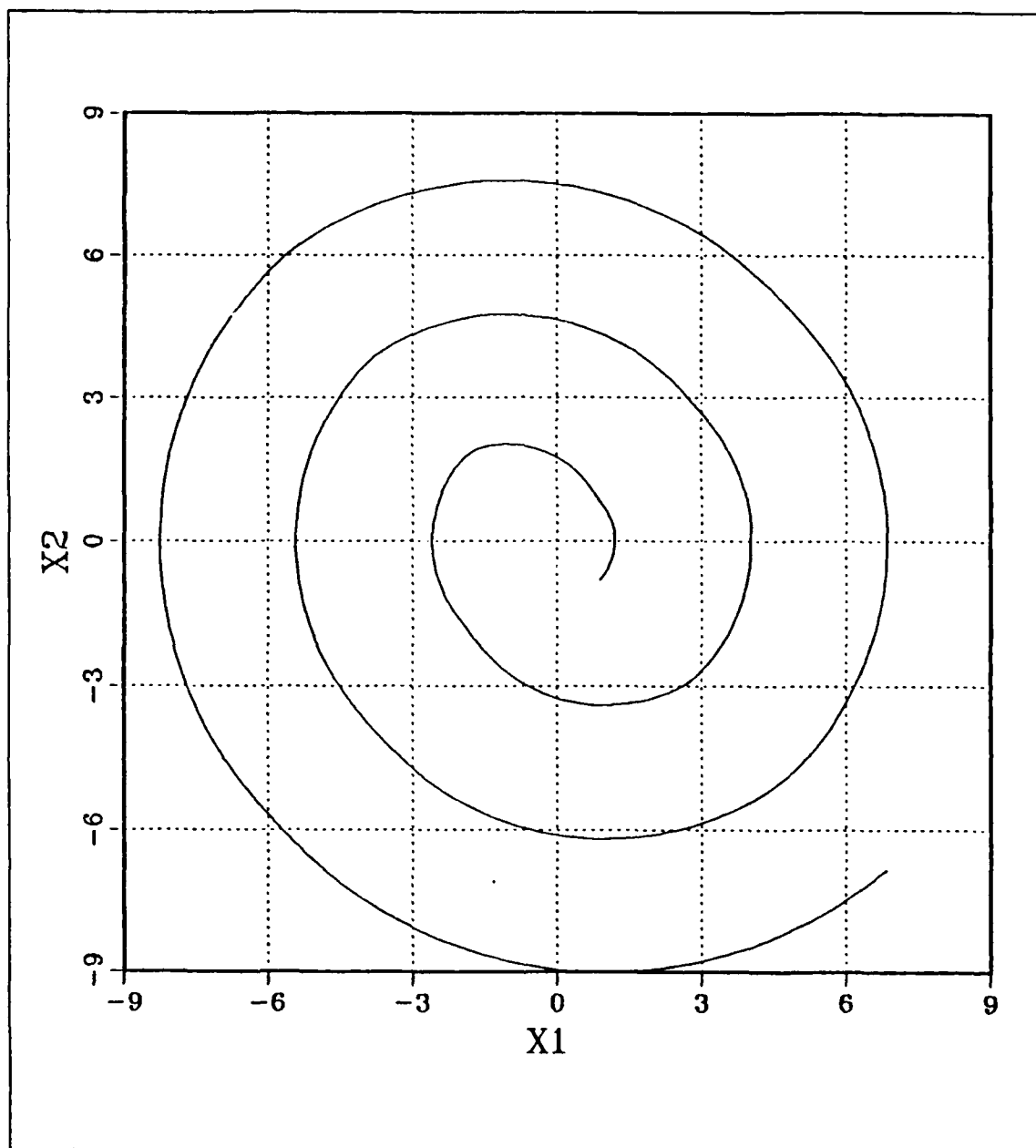$\theta = 90$ degrees
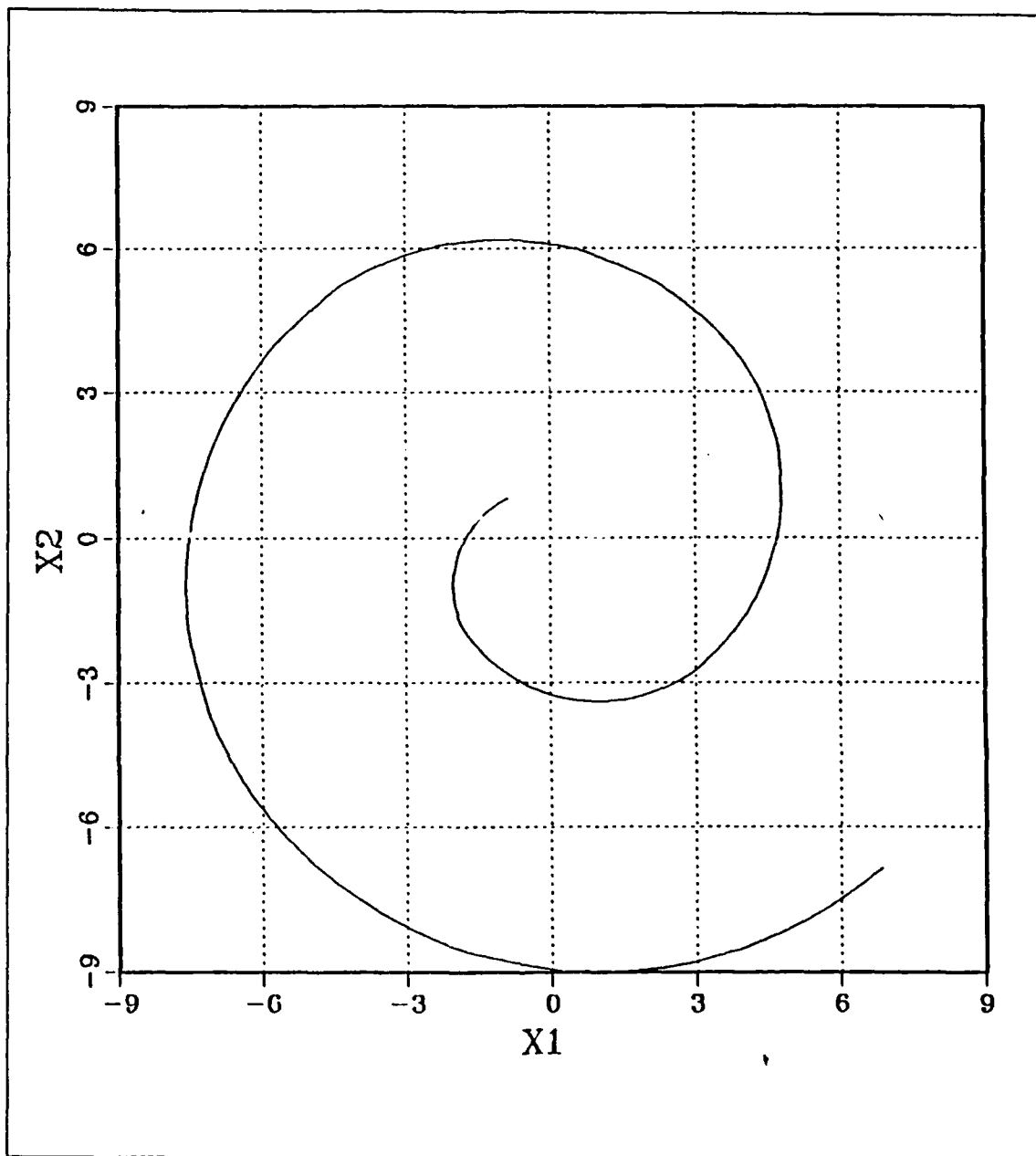
Figure 13. Two Controllers - Minimum Fuel/Time for the Far Field: Initial Condition (6.844,-6.844); $\theta = 90$ degrees
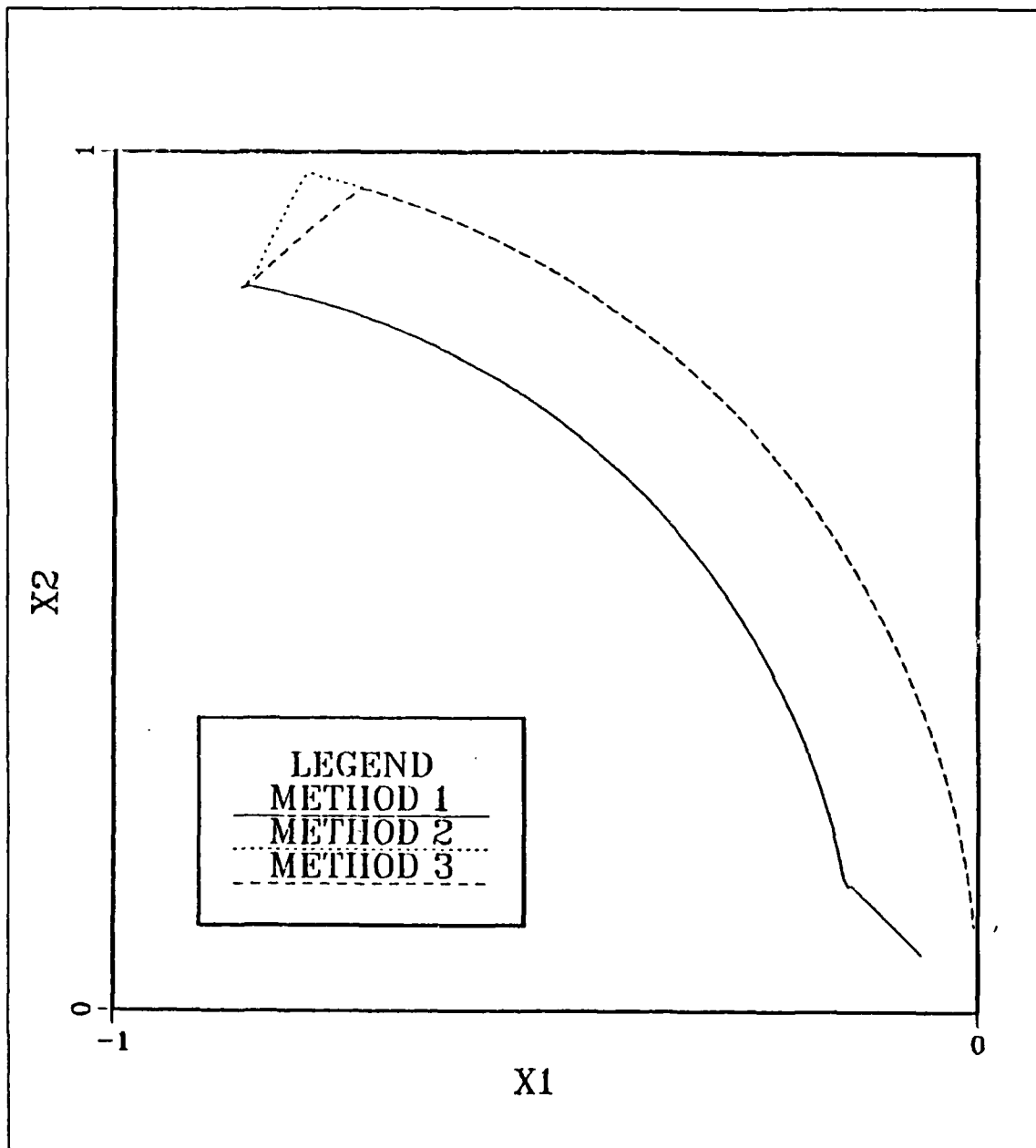
Figure 14. Two Controllers - Minimum Fuel/Time for the Near Field: $\theta = 90$ degrees

The different control regions for the dual control case where $\theta < 90$ degrees (Case 2) is shown in Figure 15. There are four regions where both controllers are simultaneously on. The switching curve for these minimum time regions is shown in

Figure 10. All other regions comply with the minimum fuel time switching curves as given in Figure 12 using Method 3, from Table 3, for the controller assignment for the shaded regions. Case 2 is a much more complicated scenario than previously encountered. There are now eight regions of controversial controller assignment. This case will NOT be used in this study, but is mentioned in order to complete the possible scenarios for spinning satellite minimum fuel time control problems.
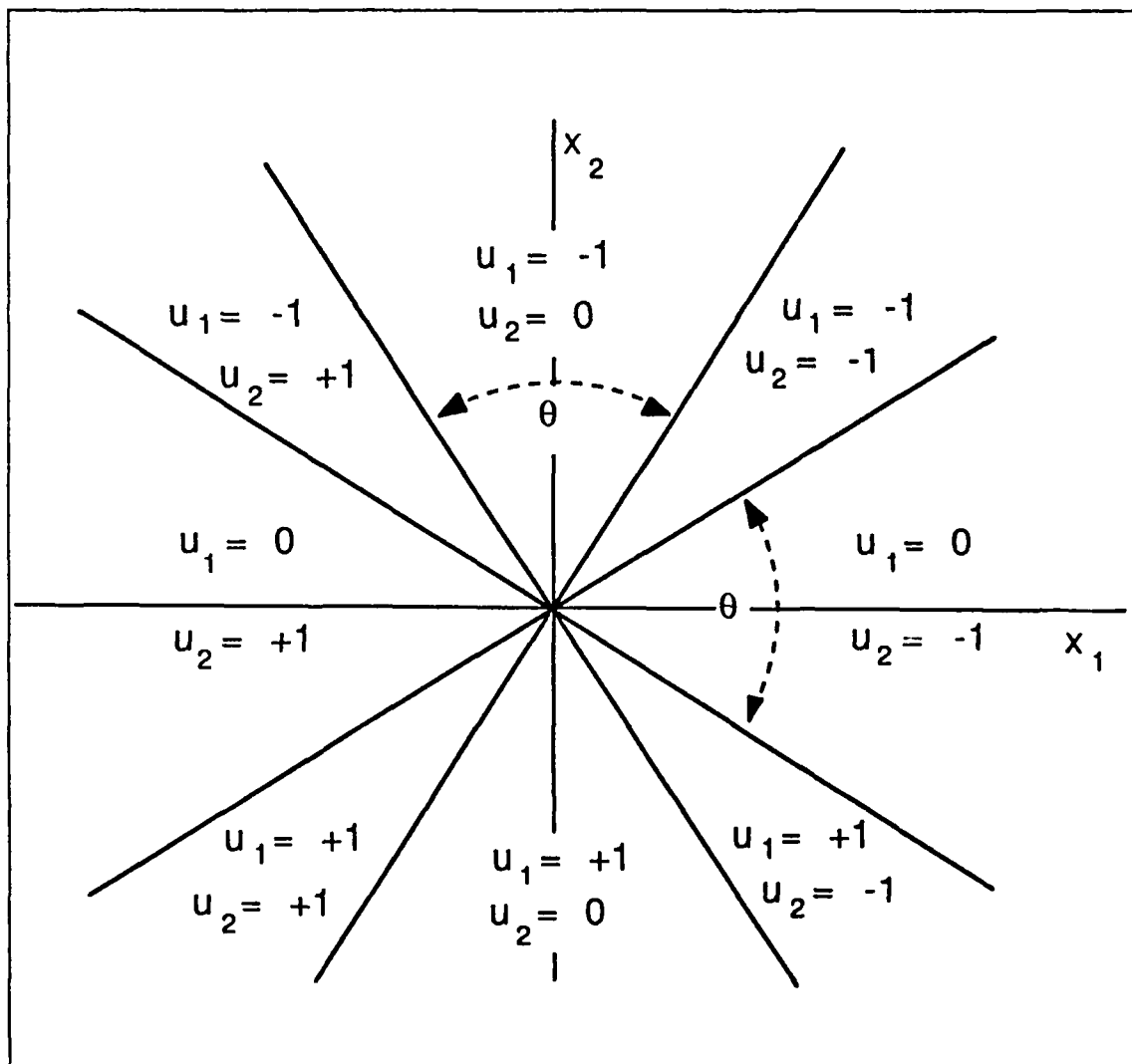
Figure 15.   Controller Assignment for Two Controllers - Case 2: $\theta < 90$ degrees

3.   Analysis

The switching curves for the single controller with a 100% duty cycle, where u_1 is always on, is shown in Figure 4. This is the minimum time problem. Figure 6 has

23

the switching curves for the single controller with the deadzone angle, θ, equal to 90 degrees. The switching curves for the dual controller with a 50% duty cycle for EACH of the two controllers, u_1 and u_2, are shown in Figure 12. No more than one of the controllers is ever on at any given time. Table 5 compares the results of these different configurations.

Table 5. SINGLE VS DUAL CONTROLLER RESULTS

	Time to End Condition	Fuel Consumed
Single Control: Minimum Time $\theta = 0$ degrees	14.73	14.73
Single Control (Method 3) $\theta = 90$ degrees	20.17	10.75
Dual Control (Method 3) $\theta = 90$ degrees	10.75	10.60

The end condition for all cases is described by a circle of radius .01 about the axes' origin. In comparing the minimum time, single control system to Method 1 of the two control minimum fuel time configuration with $\theta = 90$ degrees, not listed in Table 5, Method 1 achieves 26.6% fuel savings. A fuel savings of 28% is achieved when comparing Method 3. Method 3 improves fuel efficiency since there is actually less than a 50% duty cycle because the controllers in the four shaded cusp regions (Figure 12 on page 20) are turned off. Table 5 confirms that with regard to both time and fuel efficiency, the dual controller is by far the preferred system.

What minimum fuel time configuration of the dual control system should be employed? This depends on the parameter λ, the constant that provides a trade-off between fuel usage and time duration of the system being driven from some initial condition to a designated end condition. The following chapter develops a possible solution to this problem.

IV. NUMERICAL ANALYSIS

The objective is to find a functional relationship between θ, the deadzone angle, and λ, the fuel time trade-off parameter. In order to facilitate this Equation 18 is rewritten as the following simplified equation:

$$J = T + \lambda F \tag{27}$$

where T, the total system time, is

$$T = \int_{t_o}^{t_f} dt = t_f - t_o \tag{28}$$

and F, the fuel used during the evolution, is defined as

$$F = \int_{t_o}^{t_f} \sum_{i=1}^{2} |u_i(t)| \, dt \tag{29}$$

The system runs until the end condition criteria is met as defined by

$$x_1^2 + x_2^2 \le 0.01 \tag{30}$$

A. NUMERICAL DATA

The following steps describe how the data for relating λ and θ is obtained:

1. CHOOSE an initial condition for the dual control program simulation. (The point $x_1 = 6.844$ and $x_2 = -6.844$ is chosen for the first run since this is the initial condition used in all of the simulations presented in the previous chapter.)

2. RUN the dual controller fuel time simulation for varying angles of θ. (An increment of two degrees on the interval from 90 to 180 degrees is used for the first run presented below.) OBTAIN the values for T and F for each angle of θ.

3. CHOOSE a value of λ. COMPUTE the values of J for the various deadzone angles. (A plot of θ versus J for $\lambda = 15$ is shown in Figure 16. Observe the many local minima.) RECORD the value of θ and λ that gives the minimum computed value for J.

4. REPEAT from 3. with a new value of λ.

25
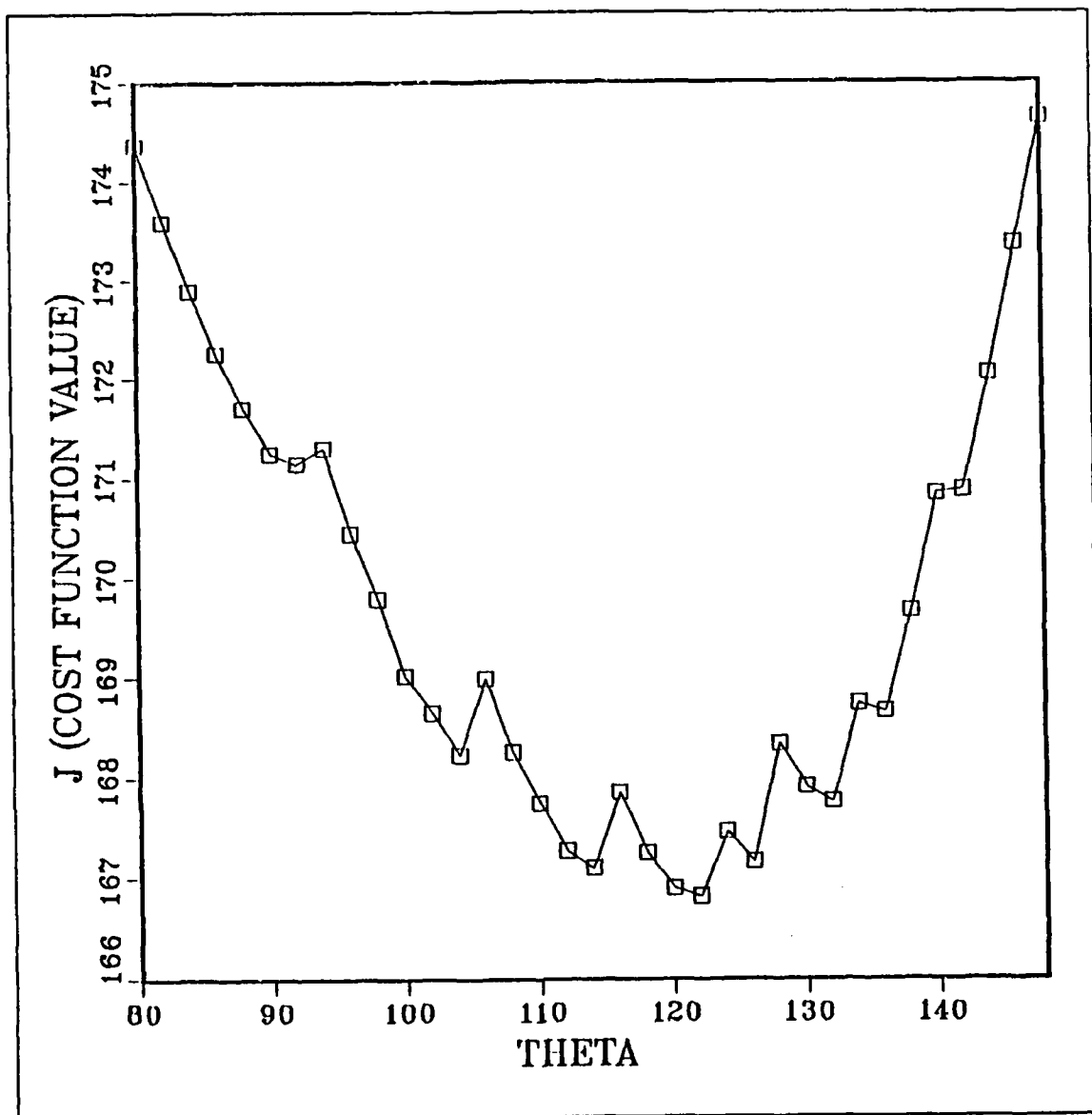
Figure 16. The System Cost Function

Figure 17 shows the first run curve of λ versus θ. It displays an exponential character. Additional values of λ are iterated using the above steps. Figure 17 is amplified with the new data points and the scale is change to semilog. See Figure 18.
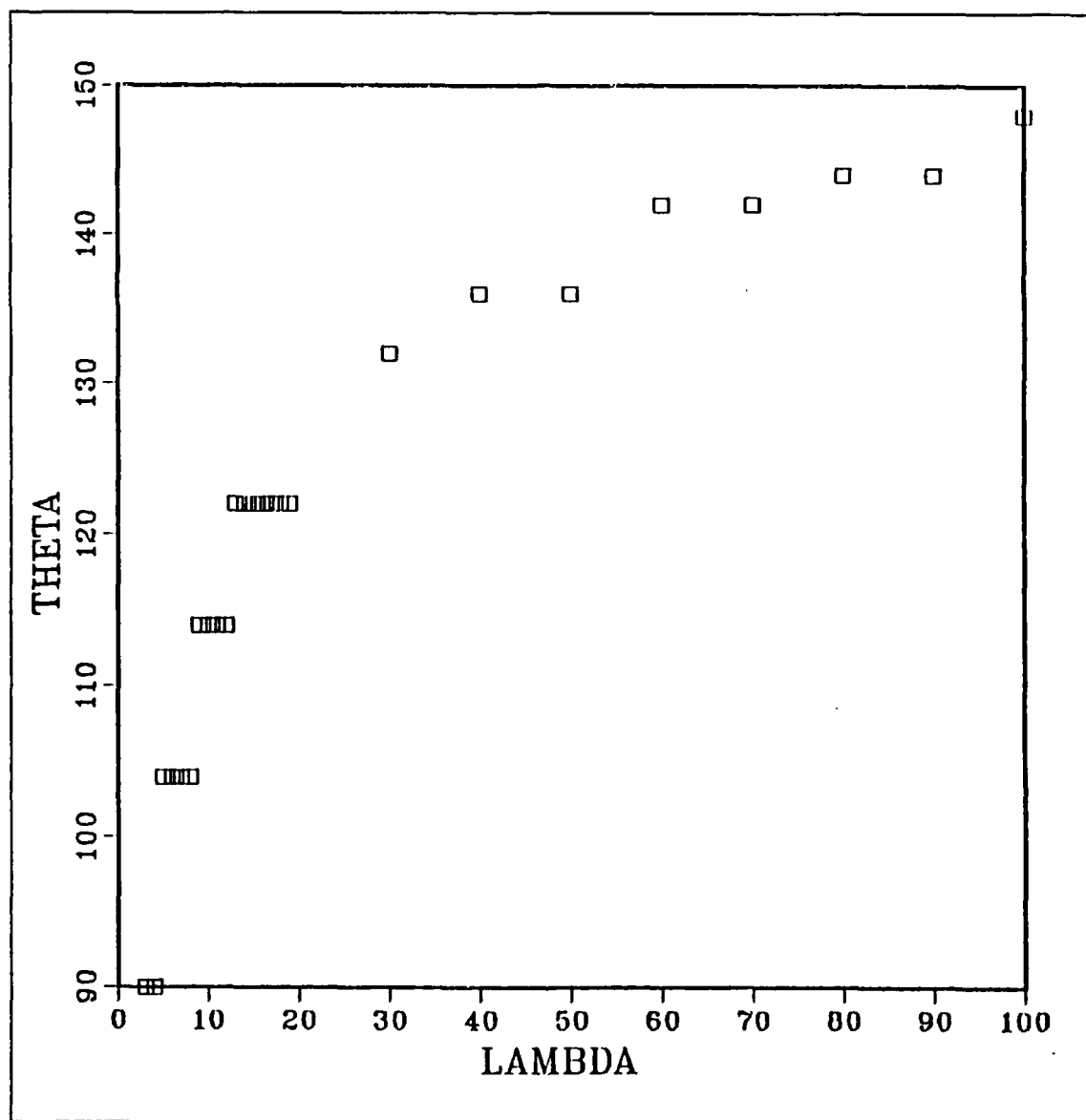
Figure 17.   Linear Plot Relating the Deadzone Angle to the Fuel/Time Tradeoff
Parameter
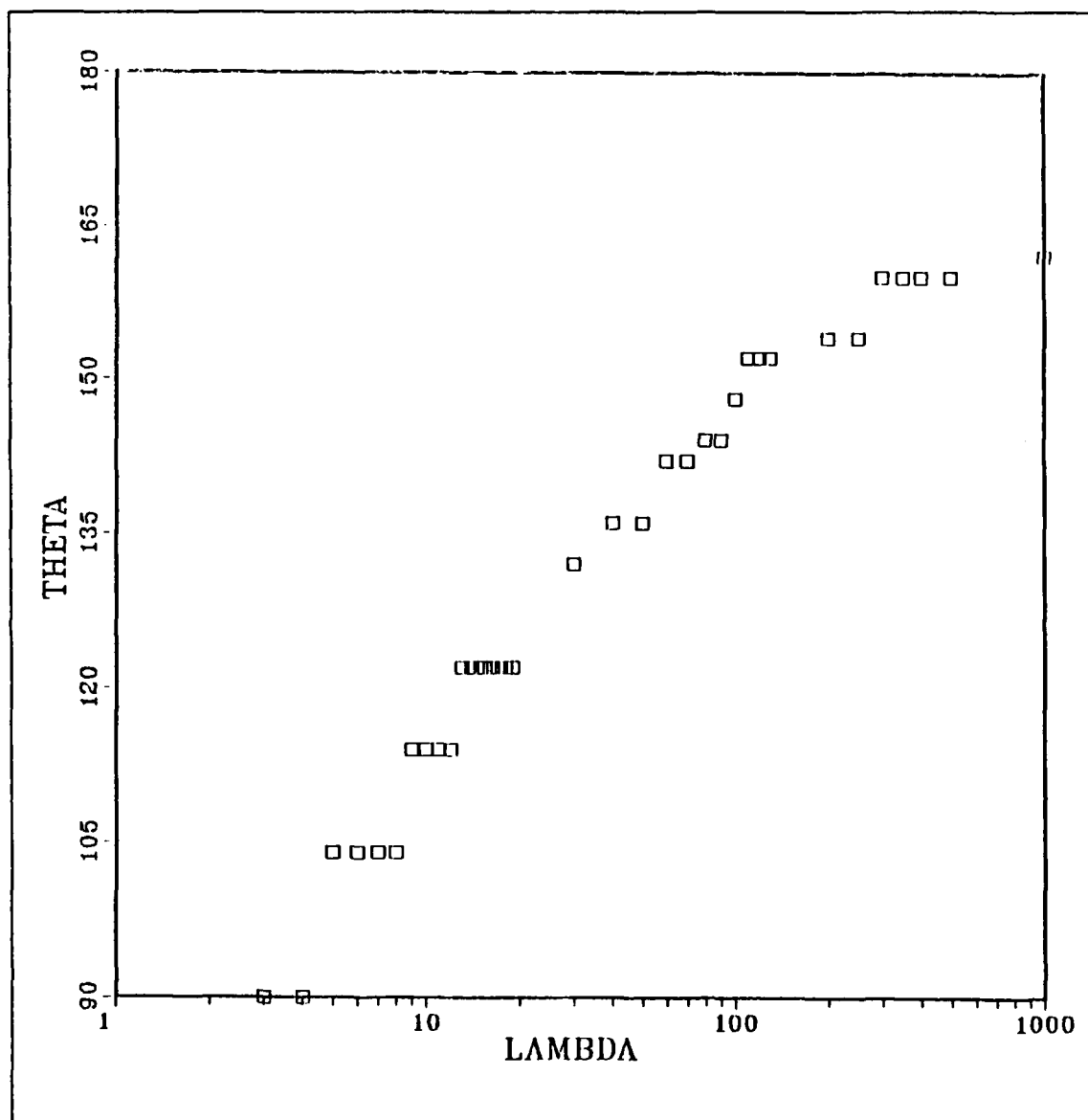
Figure 18.   Semilog Plot Relating the Deadzone Angle to the Fuel/Time Tradeoff
Parameter

B. LINEAR REGRESSION

To determine if the relation between λ and θ can truly be considered exponential in
character, a linear regression is accomplished using a commercial computer package
called Minitab. First, some background is provided.

Assume for the moment that the relation between λ and θ is exponential in nature. Then, this relation can be expressed by the following equation:

$$\lambda = Ae^{B\theta} \tag{31}$$

where A and B are both constants.

Taking the natural logarithm of both sides of Equation 31 results in the following new equation:

$$\ln \lambda = \ln A + B\theta \tag{32}$$

A linear regression of the data can now be accomplished. **Minitab** computes the following constants for the first run simulation data with the initial condition (6.844, -6.844): $\ln A = -5.6465$ (or $A = 0.00353$) and $B = 0.070698$.

The goodness of fit is illustrated through the linear correlation coefficient r for the number of data points, n, and is defined as:

$$r = \frac{n\left(\sum_i (\ln \lambda_i)\theta_i\right) - \left(\sum_i \theta_i\right)\left(\sum_i (\ln \lambda_i)\right)}{\sqrt{n\left(\sum_i \theta_i^2\right) - \left(\sum_i \theta_i\right)^2} \sqrt{n\left(\sum_i (\ln \lambda_i)^2\right) - \left(\sum_i \ln \lambda_i\right)^2}} \tag{33}$$

where $1 \leq i \leq n$. The linear correlation coefficient is always between the values of -1 and $+1$. Values of r close to -1 and $+1$ indicate a strong linear relationship between the variables θ and $\ln \lambda$, which means that the equation is useful in making predictions for a value of λ based on a value of θ or vice versa.

The value for r based on the data from the initial condition $(6.844, -6.844)$ is 0.977.

Numerical data is obtained for a new initial condition $(5, 0.001)$ using the same procedure steps outlined above. The values for A, B and r are: $A = 0.00236$, $B = 0.073611$ and $r = 0.975$.

V. CONCLUSIONS

This study provides simulation prog .ms for incorporating an optimized control system for a spinning satellite. The developed models support the theory that there is greater fuel efficiency using a dual control rather than a single control configuration. Additionally, fuel can be conserved by designing the system response for the maximum time permissible for completion of the evolution.

The software requited for this optimal control design is simple, can be easily implemented and will require very little computer memory. This will allow the ORION to operate autonomously while efficiently using the limited onboard fuel reserves.

APPENDIX A.   PROGRAM FOR ONE MINIMUM TIME CONTROLLER

This appendix shows the IODE program used for the minimum time single control-ler.   IODE is an interactive ordinary differential equations package that runs on the VM/CMS time sharing system.   It was developed at the Naval Postgraduate School by Roger R. Hilleary.

```
VARIABLES & INITIAL CONDITIONS:
X1 = 6.844000000                                         ;X1 = phi dot
X2 = -6.844000000                                        ;X2 = psi dot
F = .0                                                    ;F = fuel
T = .0                                                    ;T = time


SPECIAL FUNCTIONS:
END = X1**2 + X2**2                                       ;end condition
                                                         ;criteria

NEAR = ABS(X1)                                            ;absolute value of
                                                         ;X1

U1 = ODEIF(X2,0,1,-1)                                     ;if X2 is above the
                                                         ;x-axis U1=-1, else
                                                         ;U1=+1

U2 = ODEIF(-X1*ABS(X1)/2+ X1 + X2*ABS(X2)/2,0,1,-1)      ;if point (X1,X2)
                                                         ;above the switch-
                                                         ;ing curve U2=-1,
                                                         ;else U2=+1

U = ODEIF(NEAR,2,U2,U1)                                   ;if the absolute
                                                         ;value of X1 is
                                                         ;less than 2, then
                                                         ;U=U2, else U=U1
                                                         ;U = control input



DERIVATIVES:
D(X1 /D(T ) = =                                           ;X1 dot = X2
    X2
D(X2 /D(T ) = =                                           ;X2 dot = -X1 + U
    -X1 + U
D(F /D(T ) = =                                            ;F dot = abs(U)
    ABS(U)
```

```
OUTPUTS:
   TITLE: MINIMUM TIME PROBLEM
   TABULATE: T          X1        X2        F         U
      AT INTERVAL    .1000000000D-01
   PLOT: X2
      AGAINST: X1
   PLOT: U        U2        U1
      AGAINST: T
   PLOT: F
      AGAINST: T      AT  INTERVAL    .1000000000


END CALCULATION WHEN END .LE.  .100000D-01
```

APPENDIX B. PROGRAM FOR MINIMUM FUEL/TIME SINGLE CONTROLLER

This appendix shows the IODE program used for the minimum fuel/time single controller. IODE is an interactive ordinary differential equations package that runs on the VM/CMS time sharing system. It was developed at the Naval Postgraduate School by Roger R. Hilleary.

```
VARIABLES & INITIAL CONDITIONS:
X1 = 6.844000000                              ;X1 = phi dot
X2 = -6.844000000                             ;X2 = psi dot
F = .0                                        ;F = fuel
T = .0                                        ;T = time




CONSTANTS:
P = 1.000000000                               ;P = tan(theta)




SPECIAL FUNCTIONS:
UNEAR = ODEIF(-X1*ABS(X1)/2+X1+X2*ABS(X2)/2,0,1,-1)   ;if point (X1,X2)
                                              ;is below or on the
                                              ;function described
                                              ;by the first
                                              ;argument of the
                                              ;ODEIF line
                                              ;then UNEAR=+1,
                                              ;else UNEAR=-1


CHECK = ODEIF(X2+P*X1,0,1,-1)                 ;if point (X1,X2)
                                              ;is below or on the
                                              ;line described by
                                              ;the first argument
                                              ;of the ODEIF line
                                              ;then CHECK=+1
                                              ;else CHECK=-1


UFAR = ODEIF(ABS(X2/X1),P,0,-X2/ABS(X2))      ;if point (X1,X2)
                                              ;is below or on the
                                              ;lines with slope P
                                              ;AND slope -P
                                              ;OR
                                              ;above lines with
                                              ;slopes P AND -P
                                              ;(lines drawn thru
                                              ;origin)
                                              ;then UFAR=0, else
                                              ;UFAR=-sign(x2)
```

33

```
UCOND = ODEIF(ABS(X1),2,0,UFAR)                          ;if magnitude of X1
                                                         ;gt 2, UCOND=UFAR,
                                                         ;else UCOND=0


U = ODEIF(ABS(UNEAR+CHECK),0,UCOND,UFAR)                 ;if point (X1,X2)
                                                         ;in REGION A or B
                                                         ;then U=UCOND, else
                                                         ;U=UFAR


END = X1*X1 + X2*X2                                       ;end condition
                                                         ;criteria



DERIVATIVES:
D(X1 /D(T ) = =                                           ;X1 dot = X2
    X2
D(X2 /D(T ) = =                                           ;X2 dot = -X1 + U
    -X1 + U
D(F /D(T ) = =                                            ;F dot = abs(U)
    ABS(U)



OUTPUTS:
    TITLE: SINGLE CONTROLLER MINIMUM FUEL/TIME
    TABULATE: T          X1        X2        F         U
       AT INTERVAL    .1000000000D-01
    PLOT: X2
       AGAINST: X1
    PLOT: U
       AGAINST: T
    PLOT: F
       AGAINST: T        AT INTERVAL    .1000000000


END CALCULATION WHEN END .LE. .100000D-01
```

APPENDIX C. PROGRAM FOR MINIMUM FUEL/TIME DUAL CONTROLLER

This appendix shows the IODE program used for the minimum fuel/time dual controller. IODE is an interactive ordinary differential equations package that runs on the VM/CMS time sharing system. It was developed at the Naval Postgraduate School by Roger R. Hilleary.

```
VARIABLES & INITIAL CONDITIONS:
X1 = 6.844000000                                    ;X1 = phi dot
X2 = -6.844000000                                   ;X2 = psi dot
F = .0                                              ;F = fuel
T = .0                                              ;T = time



CONSTANTS:
P = 1.000000000                                     ;P = tan(theta)
Q = 1.000000000                                     ;Q = tan(90-theta)



SPECIAL FUNCTIONS:
UNEAR1 = ODEIF(-X1*ABS(X1)/2+X1+X2*ABS(X2)/2,0,1,-1) ;if point (X1,X2)
                                                     ;is below or on the
                                                     ;function
                                                     ;described by the
                                                     ;first argument of
                                                     ;the ODEIF line
                                                     ;then UNEAR1=+1
                                                     ;else UNEAR1=-1

CHECK1 = ODEIF(X2+P*X1,0,1,-1)                       ;if point (X1,X2)
                                                     ;is below or on the
                                                     ;line described by
                                                     ;the first argument
                                                     ;of the ODEIF line
                                                     ;then CHECK1=+1
                                                     ;else CHECK1=-1

UFAR1 = ODEIF(ABS(X2/X1),P,0,-X2/ABS(X2))           ;if point (X1,X2)
                                                     ;is below or on the
                                                     ;lines with slopes
                                                     ;P AND -P
                                                     ;OR
                                                     ;above or on lines
                                                     ;with slopes P AND
                                                     ;-P (lines drawn
                                                     ;thru origin) then
```

35

```
                                                    ;UFAR1=0, else
                                                    ;UFAR1=-sign(X2)

UCOND1 = ODEIF(ABS(X1),2,0,UFAR1)                   ;if magnitude of X1
                                                    ;gt 2, UCOND1=UFAR1
                                                    ;else UCOND1=0

U1 = ODEIF(ABS(UNEAR1+CHECK1),0,UCOND1,UFAR1)       ;if point (X1,X2)
                                                    ;in REGION a or b
                                                    ;then U1=0 else
                                                    ;U1=UFAR1

UNEAR2 = ODEIF(X1*ABS(X1)/2-X2+X2*ABS(X2)/2,0,1,-1) ;if point (X1,X2)
                                                    ;is to the left or
                                                    ;on the function
                                                    ;decribed by the
                                                    ;first argument of
                                                    ;the ODEIF line
                                                    ;then UNEAR2=+1
                                                    ;else UNEAR2=-1

CHECK2 = ODEIF(X2-Q*X1,0,-1,1)                      ;if point (X1,X2)
                                                    ;is below or on the
                                                    ;line described by
                                                    ;the first argument
                                                    ;of the ODEIF line
                                                    ;then CHECK2=+1
                                                    ;else CHECK2=-1

UFAR2 = ODEIF(ABS(X2/X1),Q,-X1/ABS(X1),0)           ;if point (X1,X2)
                                                    ;is below or on the
                                                    ;lines with slopes
                                                    ;Q AND -Q
                                                    ;OR
                                                    ;above or on lines
                                                    ;with slopes Q AND
                                                    ;-Q (lines drawn
                                                    ;thru origin) then
                                                    ;UFAR2=-sign(X1)
                                                    ;else UFAR2=0

UCOND2 = ODEIF(ABS(X2),2,0,UFAR2)                   ;if magnitude of X2
                                                    ;gt 2, UCOND2=UFAR2
                                                    ;else UCOND2=0

U2 = ODEIF(ABS(UNEAR2+CHECK2),0,UCOND2,UFAR2)       ;if point (X1,X2)
                                                    ;in REGION c or d
                                                    ;then U2=0 else
                                                    ;U2=UFAR2

END = X1*X1 + X2*X2                                 ;end condition
                                                    ;criteria


DERIVATIVES:
D(X1 /D(T ) = =                                     ;X1 dot = X2 + U2
```

```
      X2 + U2
D(X2 /D(T ) = =                                         ;X2 dot = -X1 + U1
      -X1 + U1
D(F /D(T ) = =                                          ;F dot = abs(U1) +
      ABS(U1) + ABS(U2)                                 ;          abs(U2)


OUTPUTS:
   TITLE: DUAL CONTROLLER MINIMUM FUEL/TIME
   TABULATE: T         X1        X2       U1       U2       F
      AT INTERVAL     .1000000000D-01
   PLOT: X2
      AGAINST: X1
   PLOT: U1        U2
      AGAINST: T       AT INTERVAL      .1000000000


END CALCULATION WHEN END .LE. .100000D-01
```

LIST OF REFERENCES

1. Boyd, Austin J., *Design Considerations for the ORION Satellite: Structure, Propulsion and Attitude Control Subsystems for a Small General Purpose Satellite*, Master's Thesis, Naval Postgraduate School, Monterey, California, September 1987.

2. Agrawal, Brij N., *Design of Geosynchronous Spacecraft*, Prentice-Hall Inc., 1986.

3. Junkins, John L. and Turner, J. D., *Optimal Spacecraft Rotational Maneuvering*, Elsevier, 1986.

4. Wertz, James R., *Spacecraft Attitude Determination and Control*, D. Reidel Publishing Company, 1985.

5. Bushaw D., "Optimal Discontinuous Forcing Terms", *Contributions to the Theory of Nonlinear Oscillation, Vol. IV*, p. 29, Princeton University Press, 1958.

6. Kirk, Donald E., *Optimal Estimation*, Prentice-Hall Inc., 1970.

7. Athanassiades, Michael and Falb, Peter L., *Time-Optimal Velocity Control of a Spinning Space Body*, Report 22G-8, MIT Lincoln Laboratory, September 1962.

INITIAL DISTRIBUTION LIST

11. Commander Space and Naval Warfare Systems Command   1
   PD-80
   Washington, DC 20361-5100

12. LCDR J. L. Cunningham   2
   NAVSPASUR DET ECHO
   APO San Francisco 96287-0006